Supplementary File 15: Photography Sub Study Calculations

1) Active Extension Deficit MCP

The ordinary least squares fit gave the following estimated relationship between difference and magnitude.

$$\mathbb{E}[D] = -5.54 - 0.16A$$

where *D* denotes the differences between measurement types, and *A* denotes the mean of the two active extension deficit measurements. We then obtained an estimate of the variance around the estimated expected difference by regressing the absolute residuals from this first stage on the estimated magnitudes of the available pairs of measurements. An ordinary least squares fit gave the following estimates

$$\mathbb{E}[R] = 13.09 - 0.02A$$

where R denotes the absolute residuals from the first stage, and A is as before. Assuming the residuals from the first stage are approximately Gaussian, the estimated expected difference between methods and 95% limits of agreement are given by:

$$-5.54 - 0.16A \pm \Phi^{-1}(0.975) \sqrt{\frac{\pi}{2}} [13.09 - 0.02A]$$
$$= -5.54 - 0.16A \pm (32.15 - 0.06A)$$

2) Active Extension Deficit PIP

The estimated mean difference and 95% limits of agreement (as a function of the magnitude of the measurements) are:

$$9.69 - 0.05A \pm \Phi^{-1}(0.975) \sqrt{\frac{\pi}{2}} [6.36 + 0.13A]$$
$$= 9.69 - 0.05A \pm (15.63 + 0.32A)$$

where *A* denotes the magnitude of extension deficit (as estimated by the mean of the two measurements).

3) Flexion MCP/PIP

The model for the expected difference between methods as a function of magnitude used a three knot restricted cubic spline to model magnitude (with knots placed at the 10th, 50th and 90th percentiles of the observed values). Variance around this relationship was modelled conditional on a linear term for magnitude.

4) Flexion DIP

The estimated mean difference and 95% limits of agreement (as a function of the magnitude of the measurements) are:

$$-16.49 + 0.24A \pm (41.67 - 0.29A)$$

where A denotes the mean magnitude of the flexion measurements

5) Active Extension Deficit - MCP

The model for the expected difference as a function of magnitude used a five knot restricted cubic spline to model magnitude (with knots placed at the 5th, 27.5th, 50th, 72.5th and 95th percentiles of the observed values). Variance around this relationship was modelled conditional on a linear term for magnitude.

6) Active Extension Deficit - PIP

The following estimated expected difference and 95% limits of agreement were obtained;

$$4.48 \pm (20.49 + 0.14A)$$

where *A* is the estimated magnitude of the measurement.

7) Active Extension - Reference Joint

The first to estimate the expected difference between methods conditional on a linear term for the estimated magnitude of extension deficit, and the second to estimate the agreement between the two methods of measurement (again conditional on a linear term for magnitude) once the relationship between difference and magnitude is accounted for. This gave the following estimated mean difference and 95% limits of agreement:

 $-28.97 + 0.37A \pm (25.08 + 0.04A)$